

# Generalized Edwards Transformation and Principle of Permutation Invariance

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**Abstract** The relativity of simultaneity is generalized by using the concept of synchronization gauge. The Lorentz transformation is derived without the postulate of the universal limiting speed, and a generalized Edwards transformation is obtained by using the principle of permutation invariance (covariance). It is shown that the existences of the one-way universal limiting speed (in the Lorentz transformation) and the constancy of the two-way average speed of light (in the Edwards transformation) are the necessary consequences of the principle of permutation invariance that is consistent with the postulate of relativity. The connection between the Edward transformation and the general coordinate transformation is discussed, and based on this, we find that the physical meaning of the Edwards parameter, which indicates anisotropy of the speed of light, is a gravitomagnetic potential of the spacetime.

**Keywords** Edwards transformation · Principle of permutation invariance · Standard synchrony · Synchronization gauge

## 1 Introduction

In classical mechanics, field theory and electrodynamics [1], the Lorentz transformation is derived by using Einstein's two postulates. In fact, the Lorentz transformation can also be obtained without the second postulate, i.e., the existence of a universal limiting speed (constancy of the speed of light). In other words, the universal limiting speed can be derived from the purely logical deduction (related to the principle of relativity). Historically, such a viewpoint was suggested by some authors [2–5]. In this paper, an alternative way based on a

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so-called principle of permutation invariance (covariance) is proposed to confirm our belief that a priori assumption of a universal limiting speed is not necessary in the derivation of spacetime coordinate transformations (e.g., the Lorentz and Edwards transformations) [6], and on the contrary, the existence of the universal limiting speed is actually a necessary consequence of the postulate of relativity. Furthermore, by using the principle of permutation invariance, we generalize the Lorentz and Edwards transformations to a more general form that agrees with the postulate of relativity. We show how far such a generalization can go under the condition that the generalization fulfills the principle of relativity.

We first discuss two concepts of observable quantities and clock synchronization, and then analyze possibility of arbitrariness for definition of simultaneity that would lead to various coordinate transformations (e.g., the Edwards transformation), where the Lorentz transformation is simply a special one that corresponds to Einstein's simultaneity (standard synchrony).

It is worth emphasizing that the one-way speed of light is not an observable quantity, since when we measure the one-way speed between two distant points, the time measurement involves two clocks, which should have been synchronized first and then placed at these two points, respectively [6]. However, this needs a priori assumption of a universal limiting speed (e.g., a finite, constant speed or an infinitely large velocity) [7]. Someone may suggest that we can make use of the means of synchronizing clocks by *slow transport*, i.e., first one sets two clocks at one point (say, point A), and then carries one clock very slowly to another point (say, point B) [8]. But such a synchronization might be affected by the effects taking place when carrying the clock from point A to point B. If the effect of motion on the translated clock has been known to us, then this method might be feasible [6]. But we have so far never had ideas about how the motion would affect the translated clock. This means that the postulate of constancy of the one-way speed of light lack the physical foundations [6]. However, the two-way average speed of a light from point A travelling to point B and by reflection, back to A, is an observable quantity since here only one clock is involved, and the procedure of clock synchronization is not required. In other words, the experiments (such as Michelson-Morley experiment), which were said to verify the postulate of constancy of the speed of light, actually demonstrates that only the two-way speed of a light (in a closed path of given length) is invariant, and the constancy of the one-way speed of light has so far never been examined in experiments [6]. The Edwards transformation, which was suggested in 1963 based on the postulate of the constancy of the two-way average speed of light, is a generalization of the Lorentz transformation. In this transformation, the speed of light in free vacuum is anisotropic, but the average speed of light in a two-way (round-trip) process is a constant number (independent of reference frames). Although there has so far never been any experimental evidences for the anisotropy of the speed of light, the Edwards transformation is a self-consistent coordinate transformation from the point of view of both physical and mathematical logic.

In the above, we pointed out that there exist no absolute (unique) definitions of simultaneity. In what follows, we discuss in detail the possibility of the arbitrariness of simultaneity within the framework of special relativity. The relativity of simultaneity may be the most important quintessence in special relativity. In general, the special relativity is conveyed often by using Einstein's definition of simultaneity (usually called "standard synchrony"). But this is not the unique existing example of relativity of simultaneity, because the principle of causality allows the existence of other alternative definitions of simultaneity [9, 10]. In an Einsteinian universe, no causal influence can travel faster than the speed of light in vacuum. However, Reichenbach suggested that there is no reason to rule out the possibility of arbitrarily fast causal influences, which would then be able to single out a unique event at

point A that would be simultaneous with the event at point B, because the unique standard to determine the time sequence is such that all the consequences follow the causations [9]. If, for example, a pulse travels from point A at time  $t_1$  to point B and by reflection, back to point A at time  $t_3$ , then the time  $t_2$  when the pulse arrives at point B (measured by the clock at point B) can be said to be simultaneous with the time  $t_1 + \epsilon(t_3 - t_1)$  (measured by the clock at point A), where  $\epsilon \in [0, 1]$ . It can be verified that as long as  $\epsilon \in [0, 1]$ , such a  $t_2$  will not violate the principle of causality [9]. In particular, Einstein's definition of simultaneity (standard synchrony) corresponds to a special case, where  $\epsilon = 1/2$ . Here Einstein's standard synchrony is equivalent to the requirement that the one-way speeds of the light be the same on the two segments of its round-trip journey between points A and B. Historically, someone argued that Einstein's definition of simultaneity is the only possible choice. But in fact, within the framework of special relativistic physics, other alternative choices ( $\epsilon \neq 1/2$ ), although perhaps less convenient, are indeed possible. The value of  $\epsilon$  depends upon the means of clock synchronization. If we have a pulse signal with infinite velocity, then we can determine the value for  $\epsilon$ . But actually, there might exist no such a pulse signal, and it is therefore impossible within the framework of the special relativity for any synchrony methods to result in fixing any particular value of  $\epsilon$  to the exclusion of any other particular values [11]. In other words, the one-way speed of light cannot be determined and is *not* an observable quantity. This means that the definition of simultaneity is in a sense arbitrary, and that Einstein's simultaneity is one of the most simplest ones among all the definitions of simultaneity based on various means of clock synchronization [12]. Different means of clock synchronization can yield different effects on the quantities such as the one-way velocity and the simultaneity, which are not directly observable. But they will not affect all the observable quantities such as the two-way average speed of light (in this sense, the Edwards transformation is equivalent to the Lorentz transformation. We will show in this paper that the Edwards transformation is different from the Lorentz transformation only by a global coordinate transformation).

As there are various possible means of clock synchronization, and some alternative definitions of simultaneity different from Einstein's standard synchrony do not violate the principle of causality, we can adopt a concept of *synchronization gauge* to indicate the arbitrariness in the definition for the relativity of simultaneity. We show that the essence of the synchronization gauge is just the general coordinate transformation. In the present paper, we use a so-called principle of permutation invariance (and covariance), which is consistent with the postulate of relativity, to derive the Lorentz and Edwards transformations (including the generalized form of the Edwards transformation). The postulate of the universal limiting speed is no longer required in the derivation of the Lorentz transformation. Instead, it is a theoretical consequence of the principle of permutation invariance. In the derivation of the Edwards transformation, the constancy of the two-way average speed of light in a closed path can be derived by using the procedure of permutation operation. The connection between the Edwards transformation and the general coordinate transformation is considered with special emphasis on the physical meaning of the Edwards parameter that indicates the anisotropy of the one-way speed of light as well as the spacetime.

## 2 Derivation of the Lorentz Transformation

### 2.1 Coordinate Transformation without Light

In the derivation procedure of the Lorentz transformation, we first consider the equation of motion of an ordinary test particle (rather than of a photon), and then set zero for the derivative of the transformation coefficient with respect to the particle velocity (the transformation

coefficient should be independent of the motion of the test particle). Thus a preliminary form of the coordinate transformation would be achieved.

We are now in a position to study the coordinate transformation without light. Consider two inertial reference frames ( $K$  and  $K'$ ) with spacetime coordinates  $(x, y, z, t)$  and  $(x', y', z', t')$ , respectively. These two frames move relative to each other at a relative velocity  $v$  along  $\hat{x}$ -direction. Note that in standard derivation of the Lorentz transformation, the most simple linear coordinate transformation may be as follows

$$\begin{cases} x' = k(x - vt), \\ x = k(x' + vt'). \end{cases} \quad (1)$$

Here, for convenience, we discuss the transformation of the  $1 + 1$  dimensional spacetime only. Assume that a test particle is moving in frame  $K$  with velocity  $u$  and in primed frame  $K'$  with velocity  $u'$ . The equations of motion of this particle in frames  $K$  and  $K'$  are therefore  $x = ut$  and  $x' = u't'$ , respectively, provided that the initial location at  $t = t' = 0$  in frames  $K$  and  $K'$  are coincident at the origins of  $K$  and  $K'$ . Substitution of these two equations into (1) yields

$$\frac{1}{k^2} = \left(1 - \frac{v}{u}\right) \left(1 + \frac{v}{u'}\right). \quad (2)$$

Since the coordinate transformation coefficient  $k$  is independent of the velocities  $u$  and  $u'$  of the test particle, one can have

$$\frac{d\frac{1}{k^2}}{du} = 0. \quad (3)$$

From (3), one can obtain the following equation

$$\frac{du'}{du} = \frac{u'^2 + u'v}{u^2 - uv}, \quad (4)$$

the solution to which is of the form

$$u' = \frac{u - v}{1 + \lambda u}, \quad (5)$$

where  $\lambda$  is a parameter that is independent of the variable  $u$ . Inserting expression (5) into (2), one can obtain

$$k^2 = \frac{1}{1 + \lambda v}. \quad (6)$$

It follows from (1) and (6) that the preliminary form of the spacetime coordinate transformation is

$$\begin{cases} x' = \frac{1}{\sqrt{1 + \lambda v}}(x - vt), \\ t' = \frac{1}{\sqrt{1 + \lambda v}}(t + \lambda x), \end{cases} \quad (7)$$

and the inverse transformation is

$$\begin{cases} x = \frac{1}{\sqrt{1 + \lambda v}}(x' + vt'), \\ t = \frac{1}{\sqrt{1 + \lambda v}}(t' - \lambda x'). \end{cases} \tag{8}$$

It should be noted that the inverse transformation can also be obtained by the permutation operation, which will be discussed in detail when deriving the Edwards transformation. Apparently, transformations (7) and (8) are not explicit forms, where the  $\lambda$  parameter will be determined in what follows.

### 2.2 Coordinate Transformations among Three Inertial Frames of Reference

In an attempt to determine the  $\lambda$  parameter involved in (7) and (8), we introduce a third reference frame, and then discuss the completeness condition of the transformations. The third reference frame is  $K''$  that is moving relative to frame  $K'$  with a relative velocity  $\omega$  in the positive  $\hat{x}$ -direction. Thus there are three coordinate transformations among these three inertial reference frames  $K$ ,  $K'$  and  $K''$ . In order to distinguish the  $\lambda$  parameters involved in the transformations, (7) (the spacetime coordinate transformation from  $K$  to  $K'$ ) can be rewritten as

$$\begin{cases} x' = \frac{1}{\sqrt{1 + \lambda_1 v}}(x - vt), \\ t' = \frac{1}{\sqrt{1 + \lambda_1 v}}(t + \lambda_1 x). \end{cases} \tag{9}$$

Assume that the space and time coordinate of a point in frame  $K''$  is  $(x'', t'')$ . Then the second transformation (from  $K'$  to  $K''$ ) is given by

$$\begin{cases} x'' = \frac{1}{\sqrt{1 + \lambda_2 \omega}}(x' - \omega t'), \\ t'' = \frac{1}{\sqrt{1 + \lambda_2 \omega}}(t' + \lambda_2 x'). \end{cases} \tag{10}$$

Substituting (9) into (10), we can obtain the third transformation (from  $K$  to  $K''$ )

$$\begin{cases} x'' = \frac{1 - \lambda_1 \omega}{\sqrt{1 + \lambda_2 \omega} \sqrt{1 + \lambda_1 v}} \left( x - \frac{v + \omega}{1 - \lambda_1 \omega} t \right), \\ t'' = \frac{1 - \lambda_2 v}{\sqrt{1 + \lambda_2 \omega} \sqrt{1 + \lambda_1 v}} \left( t + \frac{\lambda_1 + \lambda_2}{1 - \lambda_2 v} x \right). \end{cases} \tag{11}$$

Note that all the transformation formulae between these three inertial reference frames should be written in the same form (and hence all physical phenomena in different reference frames occur in an identical manner). Thus, in accordance with the transformation formula (7), (11) should be rewritten in the form

$$\begin{cases} x'' = \frac{1}{\sqrt{1 + \Lambda V}}(x - Vt), \\ t'' = \frac{1}{\sqrt{1 + \Lambda V}}(t + \Lambda x), \end{cases} \tag{12}$$

where  $V$  denotes the relative velocity between frames  $K$  and  $K''$ . Comparing (12) with (11), we can obtain the expressions for parameters  $V$  and  $\Lambda$ :

$$V = \frac{v + \omega}{1 - \lambda_1 \omega}, \quad \Lambda = \frac{\lambda_1 + \lambda_2}{1 - \lambda_2 v}. \quad (13)$$

Moreover, the following relation

$$\frac{1 - \lambda_1 \omega}{\sqrt{1 + \lambda_2 \omega} \sqrt{1 + \lambda_1 v}} = \frac{1 - \lambda_2 v}{\sqrt{1 + \lambda_2 \omega} \sqrt{1 + \lambda_1 v}} \quad (14)$$

should also be fulfilled. This means that  $1 - \lambda_1 \omega = 1 - \lambda_2 v$ , or

$$\frac{\lambda_1}{v} = \frac{\lambda_2}{\omega}. \quad (15)$$

Keep in mind that the terms on the left- and right-handed sides in (15) are the parameters corresponding to their respective transformations (9) and (10). The two terms should be independent of each other. Therefore, they must be equal to a constant number, say,  $q$ , i.e.,

$$\frac{\lambda_1}{v} = \frac{\lambda_2}{\omega} = q, \quad (16)$$

and subsequently

$$\lambda_1 = qv, \quad \lambda_2 = q\omega. \quad (17)$$

Insertion of the above expressions into (13) yields

$$V = \frac{v + \omega}{1 - qv\omega}, \quad \Lambda = \frac{q(v + \omega)}{1 - qv\omega}, \quad (18)$$

where the first formula is the law for the addition of velocities, and the second formula can be rewritten as

$$\Lambda = qV. \quad (19)$$

Such a form is consistent with expression (17). Further calculation can show that both of the two terms on the left- and right-handed sides of (14) are equal to the transformation coefficient  $1/\sqrt{1 + \Lambda V}$  in transformation (12). This, therefore, means that the above formulation is self-consistent.

Thus, according to (5) and (7), the ultimate form of the spacetime coordinate transformation from  $K$  to  $K'$  is given by

$$\begin{cases} x' = \frac{1}{\sqrt{1 + qv^2}}(x - vt), \\ t' = \frac{1}{\sqrt{1 + qv^2}}(t + qvx), \end{cases} \quad (20)$$

and the law for the addition of velocities is

$$u' = \frac{u - v}{1 + qvu}. \quad (21)$$

It is apparently seen that if the parameter  $q = 0$ , then (20) and (21) are both reduced to the forms well known in Newtonian mechanics [specifically, (20) would be reduced to the Galilean transformation]. In other words, the Galilean transformation is the most simple coordinate form that fulfills the principle of relativity. However, the case of  $q \neq 0$  also satisfies the principle of relativity and is permitted to exist in physics. In the subsection that follows, we discuss the physical meanings of the parameter  $q$ , and then based on (20), show that the Galilean and Lorentz transformations are the only two self-consistent coordinate transformations in the present formulation.

### 2.3 The Existence of an Invariant Velocity

It is clear that transformation (20) has a form analogous to the Lorentz transformation, if the parameter  $q \neq 0$ . One of the most remarkable conclusions in the above formulation is that there exists an invariant velocity in formula (21) for the addition of velocities: specifically, if the invariant velocity is  $\xi$  (such an invariant velocity is a same constant in both frames  $K$  and  $K'$ , i.e.,  $u = \xi, u' = \xi$ ), then from (21) we can have

$$\xi = \frac{\xi - v}{1 + qv\xi}, \tag{22}$$

and the relation

$$q\xi^2 = -1 \tag{23}$$

can be derived. Thus the invariant velocity can be expressed in terms of the constant  $q$ , i.e.,

$$\xi = \sqrt{-\frac{1}{q}}. \tag{24}$$

By using the relation

$$q = -\frac{1}{\xi^2}, \tag{25}$$

the coordinate transformation (20) and the law (21) for velocity addition can be rewritten as

$$\begin{cases} x' = \frac{1}{\sqrt{1 - \frac{v^2}{\xi^2}}}(x - vt), \\ t' = \frac{1}{\sqrt{1 - \frac{v^2}{\xi^2}}}\left(t - \frac{vx}{\xi^2}\right), \end{cases} \tag{26}$$

and

$$u' = \frac{u - v}{1 - \frac{vu}{\xi^2}}, \tag{27}$$

where the invariant velocity  $\xi$  is involved.

Note that (26) has exactly the same form as the Lorentz transformation. Then what about the numerical value of the invariant velocity  $\xi$ ? It should be determined by the experiments. Modern experiments show that the invariant velocity  $\xi$  has the same value as the speed of light in vacuum.

In the above we derive the Lorentz transformation without the postulate of constancy of the speed of light. In contrast, such a postulate can be viewed as a necessary consequence of the derivation itself. Both the Galilean and Lorentz transformations, which correspond to the different invariant velocities, can be derived in the above formulation.

### 3 The Generalized Edwards Transformation

In the preceding section, we adopted Einstein's definition of simultaneity (standard synchrony) and assumed that the one-way velocity is an observable quantity. However, as stated in Introduction, Einstein's standard synchrony is simply one of the various possible definitions of simultaneity. In this section, we make use of the principle of permutation invariance and establish the generalized coordinate transformations, which correspond to other alternative definitions of relativity of simultaneity and agree with the postulate of relativity as well as the principle of causality. Based on this, we will suggest the generalized Edwards transformation.

#### 3.1 Transformation Coefficients Independent of the Velocities of the Test Particle

The principle of permutation invariance presented in this paper is equivalent to the postulate of relativity: specifically, the permutation operation can guarantee that the physical phenomena in all inertial frames of reference occur in an identical manner. In what follows, we show how the principle of permutation invariance works in deriving the Edwards transformation. Assume that frame  $K'$  moves relative to frame  $K$  in the positive  $\hat{x}$ -direction with velocity  $v$ , while  $K$  moves relative to  $K'$  with velocity  $-v'$ . The space coordinate transformation between frames  $K$  and  $K'$  is

$$\begin{cases} x' = k(x - vt), \\ x = k'(x' + v't'). \end{cases} \quad (28)$$

Substitution of the equation of motion (i.e.,  $x = ut$ ,  $x' = u't'$ ) of a test particle into (28) yields

$$k'k = \frac{u'u}{(u - v)(u' + v')}. \quad (29)$$

As  $k'k$  should be independent of the particle velocity  $u$ , we have  $d(k'k)/du = 0$ , and then obtain

$$\frac{du'}{du} = \frac{(u' + v')u'v}{(u - v)uv'}. \quad (30)$$

The solution to this equation is

$$u' = \frac{u - v}{\frac{v}{v'} + \lambda u}, \quad (31)$$

where  $\lambda$  is a parameter independent of the particle velocity  $u$ . Substitution of expression (31) into (29) yields

$$k'k = \frac{1}{1 + \lambda v'}. \quad (32)$$



Alternatively, by using the permutation operation ( $k' \leftrightarrow k, v' \leftrightarrow -v, \lambda \leftrightarrow \lambda'$ ) and the principle of permutation invariance, we can express  $k'k$  in terms of  $v$ , i.e.,

$$k'k = \frac{1}{1 + \lambda'(-v)}. \tag{33}$$

In the permutation operation, the symbol  $\leftrightarrow$  means the interchange of two quantities on its two sides. By comparing expression (33) with (32), one can see that if  $\lambda = \zeta v, \lambda' = -\zeta v'$ , then we have the relations  $\lambda v' = -\lambda' v = \zeta v' v$ . Here the  $\zeta$  is a parameter that is invariant under the permutation transformation. Thus, both expressions (33) and (32) can be rewritten as

$$k'k = \frac{1}{1 + \zeta v' v}, \tag{34}$$

which is invariant under the permutation operation.

According to (28), the spacetime coordinate transformation from  $K$  to  $K'$  is given by

$$\begin{cases} x' = k(x - vt), \\ t' = k\left(\frac{v}{v'}t + \zeta vx\right). \end{cases} \tag{35}$$

Under this transformation, the law for addition velocities is

$$u' = \frac{u - v}{\frac{v}{v'} + \zeta uv}. \tag{36}$$

Now let us consider the inverse transformation. By using the principle of permutation invariance ( $k \leftrightarrow k', v \leftrightarrow -v', x \leftrightarrow x', t \leftrightarrow t'$ ), we can obtain the inverse transformation

$$\begin{cases} x = k'(x' + v't'), \\ t = k'\left(\frac{v'}{v}t' - \zeta v'x'\right). \end{cases} \tag{37}$$

In the meanwhile, from (36), we can obtain the inverse transformation for the addition of velocities

$$u = \frac{u' + v'}{\frac{v'}{v} - \zeta u'v'} \tag{38}$$

by using the permutation operation ( $v \leftrightarrow -v', u \leftrightarrow u'$ ).

### 3.2 Determination of Parameters by Permutation Operation

In transformations (35) and (37) the only retained parameters (and functions), which should be determined, are  $v/v', k, k'$  and  $\zeta$ . In the following discussions, we can obtain these parameters (and functions) by using the principle of permutation invariance (covariance):

(i) It is found that the functions  $v/v'$  and  $v'/v$  should take the following forms

$$\frac{v}{v'} = \frac{1}{1 - \eta v'}, \quad \frac{v'}{v} = \frac{1}{1 + \eta v}, \tag{39}$$

where  $\eta$  is a permutation-invariance parameter that will be determined below. These two expressions satisfy the principle of permutation covariance: by using the permutation

$v \leftrightarrow -v', v' \leftrightarrow -v$ , the second expression can be rewritten as the first one, and vice versa. This means that the two expressions in (39) are the only self-consistent choices for the functions  $v/v'$  and  $v'/v$ .

(ii) It follows from (34) that the transformation coefficients  $k$  and  $k'$ , which agrees with the principle of permutation covariance, should have the following forms

$$\begin{cases} k' = \sqrt{\left(\frac{v}{v'}\right)^\sigma \frac{1}{1 + \zeta v'v}}, \\ k = \sqrt{\left(\frac{v'}{v}\right)^\sigma \frac{1}{1 + \zeta v'v}}. \end{cases} \tag{40}$$

Clearly, the second expression can also be transformed into the first one, and vice versa under the permutation transformation ( $k \leftrightarrow k', v \leftrightarrow -v'$ ).

(iii) There are various choices for the form of the parameter  $\eta$  that is invariant under the permutation operation. As one of the most simple forms,  $\eta$  can be chosen as

$$\eta = \frac{X + X'}{c}. \tag{41}$$

The physical meanings of the parameters  $X, X'$  and  $c$  will be revealed in what follows.

Let us consider a round-trip motion of a test particle in  $K$  and  $K'$ . If the particle is moving from point A to point B in frame  $K$  with a velocity  $c_+$  parallel to the positive  $\hat{x}$ -direction, and then by reflection, back to point A with a velocity  $-c_-$  parallel to the negative  $\hat{x}$ -direction. In the meanwhile, the same particle is moving from point A' to point B' in frame  $K'$  with a velocity  $c'_+$  parallel to the positive  $\hat{x}'$ -direction, and by reflection, back from point B' to point A' with  $-c'_-$  parallel to the negative  $\hat{x}'$ -direction. According to expression (36), one can obtain the relations between the to-and-fro velocities  $c_+, c_-$  and  $c'_+, c'_-$ , i.e.,

$$c'_+ = \frac{c_+ - v}{\frac{v}{c} + \zeta c_+ v}, \quad -c'_- = \frac{(-c_-) - v}{\frac{v}{c} + \zeta (-c_-) v}. \tag{42}$$

Then by using the principle of permutation invariance, one can obtain a relation

$$\begin{aligned} & v \left[ \left( \frac{1}{c'_+ c_+} - \frac{1}{c'_- c_-} \right) + \frac{X + X'}{c} \left( \frac{1}{c_+} + \frac{1}{c_-} \right) \right] \\ &= v' \left[ \left( \frac{1}{c'_+ c_+} - \frac{1}{c'_- c_-} \right) + \frac{X + X'}{c} \left( \frac{1}{c'_+} + \frac{1}{c'_-} \right) \right]. \end{aligned} \tag{43}$$

It follows that the relations

$$\begin{cases} c'_+ = \frac{c}{1 - X'}, \\ c'_- = \frac{c}{1 + X'}, \end{cases} \quad \begin{cases} c_+ = \frac{c}{1 - X}, \\ c_- = \frac{c}{1 + X} \end{cases} \tag{44}$$

satisfy (43). Here,  $X'$  and  $X$  denote the Edwards parameters, which indicate anisotropy of the speed of light. It is shown from expression (44) that the velocities  $c_+, c_-$  and  $c'_+, c'_-$  agree with the following relation

$$\frac{1}{2} \left( \frac{1}{c'_+} + \frac{1}{c'_-} \right) = \frac{1}{2} \left( \frac{1}{c_+} + \frac{1}{c_-} \right) = \frac{1}{c}. \tag{45}$$

One can verify that in (43), there exist many solutions besides (44) for  $c_+$ ,  $c_-$  and  $c'_+$ ,  $c'_-$ , and that the solution (44) that is one of the most simplest ones corresponds to the massless particles and the other more complicated solutions belong to the massive particles. If such a massless particle is photon, then the physical meaning of relation (45) is the constancy of the two-way speed of a light: specifically, the average speed of a light pulse travelling from point A to B and then back to A is an invariant quantity that is independent of the choice of the inertial reference frames. Here  $c$  denotes the two-way average speed of light in free vacuum. Clearly, since  $X$  and  $X'$  are adjustable parameters relying upon the means of clock synchronization, there is no absolute simultaneity relations, and the standard Einsteinian synchrony is simply the choice corresponding to the postulate that no causal influence can travel faster than the speed of light in vacuum.

With the help of (42) and (44) we can obtain the explicit expression for the permutation-invariance parameter  $\zeta$

$$\zeta = \frac{1}{c^2}(X^2 - 1) + \frac{X - X'}{cv}. \tag{46}$$

By using the permutation operation procedure ( $X \leftrightarrow X'$ ,  $v \leftrightarrow -v'$ ), we can obtain an alternative expression

$$\zeta = \frac{1}{c'^2}(X'^2 - 1) + \frac{X - X'}{cv'}. \tag{47}$$

It can be readily verified that the parameter  $\zeta$  in expression (47) is truly equal to that in expression (46). This means that the principle of permutation invariance presented in this paper is self-consistent. Thus, from (40), (46) and (47), the explicit expression for  $k'k$  is given by

$$k'k = \frac{1}{1 + \left[\frac{1}{c^2}(X^2 - 1) + \frac{X - X'}{cv}\right]v'v} = \frac{1}{1 + \left[\frac{1}{c'^2}(X'^2 - 1) + \frac{X - X'}{cv'}\right]v'v}. \tag{48}$$

The coordinate transformation (35) and its inverse transformation (37) with the coefficients  $k, k'$  defined as (40) can be viewed as the generalized Edwards transformations. The Edwards transformation suggested in 1963 is in fact a simple one, where the parameter  $\sigma = 1$ .

### 3.3 Reduced to the Edwards Transformation

If the parameter  $\sigma$  is taken to be 1, then the transformation coefficients  $k, k'$  in (35) and (37) are of the form

$$k = \frac{1}{\sqrt{\left(1 + \frac{v}{c}X\right)^2 - \frac{v^2}{c^2}}}, \quad k' = \frac{1}{\sqrt{\left(1 - \frac{v'}{c}X'\right)^2 - \frac{v'^2}{c^2}}}. \tag{49}$$

Thus the coordinate transformation with the constancy of the two-way average speed of light reads

$$\begin{cases} x' = k(x - vt), \\ t' = k \left[ \left(1 + \frac{X + X'}{c}v\right)t + \left(\frac{X^2 - 1}{c^2} + \frac{X - X'}{cv}\right)vx \right]. \end{cases} \tag{50}$$

It follows that the corresponding inverse transformation is

$$\begin{cases} x = k'(x' + v't'), \\ t = k' \left[ \left( 1 - \frac{X + X'}{c} v' \right) t' - \left( \frac{X'^2 - 1}{c^2} + \frac{X - X'}{cv'} \right) v' x' \right]. \end{cases} \tag{51}$$

Apparently, the inverse transformation (51) can also be obtained from (50) by using the permutation operation procedure.

#### 4 Connection Between the Edwards Transformation and the General Coordinate Transformation

The Edwards transformation (50) can be reduced to the Lorentz transformation if the Edwards parameters  $X = X' = 0$ . In the Edwards transformation, Einstein’s postulate of constancy of the one-way speed of light is replaced by the principle of constancy of the two-way average speed of light. In what follows we point out the connection between the Edwards transformation and the general coordinate transformation. The to-and-fro speeds of light in the Edwards spacetime are  $c_+ = c/(1 - X)$  and  $-c_- = -c/(1 + X)$ , respectively. It is easily seen that the two speeds,  $c_+$  and  $-c_-$ , of the light fulfill the following quadratic equation

$$(1 - X^2)u^2 - 2Xcu - c^2 = 0, \tag{52}$$

where  $u$  is defined by  $u = dx/dt$ . Apparently, the solutions of (52) are  $u_1 = c_+$ ,  $u_2 = -c_-$ . Equation (52) can be rewritten as

$$(1 - X^2)dx^2 - 2Xdxcdt - c^2dt^2 = 0, \tag{53}$$

and the matrix form for (53) reads

$$\begin{pmatrix} dx^0 & dx \end{pmatrix} \begin{pmatrix} -1 & -X \\ -X & 1 - X^2 \end{pmatrix} \begin{pmatrix} dx^0 \\ dx \end{pmatrix} = 0, \tag{54}$$

where  $dx^0 = cdt$ . It can be further rewritten as a square of spacetime interval, i.e.,

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = 0, \tag{55}$$

where  $g_{\mu\nu}$  is a spacetime metric tensor,

$$g_{\mu\nu} = \begin{pmatrix} -1 & -X \\ -X & 1 - X^2 \end{pmatrix}. \tag{56}$$

Although the tensor  $g_{\mu\nu}$  seems to be a metric of curved spacetime, it is actually a flat metric since the Edwards parameter  $X$  is constant for a certain reference frame, and therefore all the components of the Riemannian curvature tensor vanish, i.e.,  $R_{\mu\nu\alpha\beta} = 0$ . This means that the spacetime with such a line element (55) is a flat spacetime (rather than a curved one), and that the symmetric tensor (56) can be changed into a diagonal flat metric tensor by using a general coordinate transformation. It is thus believed that the Edwards transformation is different from the Lorentz transformation only by a certain general coordinate transformation (similarity transformation). Note that such a general coordinate transformation is a

global (rather than local) transformation. Therefore, the Edwards transformation is equivalent to the Lorentz transformation. The special relativity with the constancy of the two-way average speed of light predicts the same observable effects as Einstein's special relativity did.

For the Edwards spacetime, the physical meaning of the Edwards parameter  $X$  in expression (56) is analogous to a gravitational (gravitomagnetic) potential [13, 14]. However, as such a gravitomagnetic potential is a constant number, it does not lead to any force field effects for any observable physical quantities. For example, in both the Lorentz and Edwards transformations, the two-way average speed of light, which is an observable quantity, takes the same value. But the values for the one-way speed of light, which is not an observable quantity, are different in the Lorentz and Edwards transformations (the Edwards parameter in the one-way speed of light can exhibit a topological effect that is similar to the Sagnac effect). Such a difference is caused due to the so-called synchronization gauge (i.e., the arbitrariness in the definition of relativity of simultaneity).

## 5 Discussions

The one-way speed of light is not an observable quantity, since it depends on the chosen synchronization gauges. Here a question may arise: can we determine the definite and unique synchronization gauge (should such exist) for a given inertial frame? Very recently, Wang et al. suggested a new and excellent scheme to test the anisotropy of the speed of light by using the technique of phase-conjugate interferometer (phase-conjugate Sagnac experiment) [15–17]. In other words, we think such a scenario could detect the Edwards parameters. If this is true, the synchronization gauge (including the definition of simultaneity) in the inertial frame of our own could be determined. At present, this problem deserves further consideration. In this paper, we hold that there might be no absolute and unique definitions of simultaneity, because we have so far never had ideal means of clock synchronization (e.g., the infinite-speed signals). As stated in the above, Einstein's standard synchrony (the simplest definition for relativity of simultaneity) is equivalent to the constancy of one-way speed of light. But recently, Pardy suggested that the definition of simultaneity by Einstein would lead to an interesting paradox motion of the string [18]. This may mean that one should consider those more general definitions for relativity of simultaneity.

## 6 Concluding Remarks

Einstein's simultaneity (standard synchrony) is simply a special case among various possible definitions of simultaneity, which obey the principle of causality. The Lorentz transformation corresponds to one of the most simplest means of clock synchronization, and the relativity of simultaneity in Einstein's special relativity depends upon such a special choice of synchronization. The measurement of the quantities (such as the one-way speed of light), which cannot be directly observed, has a close relation to the chosen definition of simultaneity (synchronization gauge). Different definitions of simultaneity will give different results of measurement for, e.g., the one-way speed of light. But for any observable quantities (such as the two-way average speed of light), different definitions of simultaneity (and various clock synchronizations) will lead to the same measurement results.

The Lorentz transformation can be extended to the Edwards transformation when one gives up Einstein's special definition of simultaneity (standard synchrony) and adopts the

general synchronization gauge. In this paper, we showed that the Edwards transformation can also be generalized to a more general form based on the principle of permutation invariance (and covariance), which incarnates both the principle of relativity and the arbitrariness in definition of simultaneity. As there is an effective gravitomagnetic potential (the Edwards parameter), the Edwards spacetime can be considered a Riemannian spacetime rather than a Minkowski spacetime. But such a gravitomagnetic potential in the Edwards spacetime is constant (independent of the spacetime coordinates), so that there exists a global coordinate transformation, which can transform the Edwards spacetime into the Minkowski spacetime. This means that the Edwards spacetime is in fact equivalent to the Minkowski spacetime for the observable quantities (though the one-way speed of light, which is not an observable quantities, is anisotropic in the Edwards spacetime).

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